Journal of Applied Mechanics and Technical Physics, Vol. 48, No. 2, pp. 285–291, 2007

ANALYSIS OF AXISYMMETRIC PHASE STRAINS IN PLATES AND SHELLS

L. I. Shkutin

UDC 539.370

The axisymmetric strain problem for a shell in the direct phase transformation interval is formulated approximately as a nonlinear boundary-value thermoelastic problem with an implicit temperature dependence (through a phase parameter simulating the volume fraction of the new-phase crystals). The buckling problems for a circular plate and a shallow spherical dome of TiNi alloy loaded by normal pressure in the direct phase transformation interval are solved numerically. The branches of buckled equilibrium states are obtained for various values of the loading and phase parameters. It is found that the deflections increase abruptly with an increase in the phase parameter for a fixed value of the loading parameter. The evolution of the buckling modes and the phase-strain distribution along the meridian are studied.

Key words: *shape-memory alloys, phase transformations, phase strains, plates, shells, buckling, numerical analysis.*

The unique properties of shape-memory alloys are due to its thermoelastic phase transformations [1]. Cooling of a loaded specimen in the phase-transformation interval results in a phase strain whose deviator is proportional to the internal-stress deviator at a constant temperature [2]. For the subsequent heating of the specimen through the inverse-transformation interval, the phase strain acquired earlier is removed partly or completely (the shape-memory effect). Metal alloys undergoing phase transformations under thermal cycling are used mainly in the design of thermosensitive structural elements. The shape-memory effect is the most pronounced for thin-walled elements made of these alloys.

Over the last years, certain progress has been made in constructing mathematical equations for modeling the phase-transformation and shape-memory effects. In the present paper, the micromechanical constitutive relations proposed and substantiated in [2, 3] are used. In [4], these equations were applied to the linear buckling problem for a rectangular plate in the direct-transformation interval. An analysis of the nonlinear problems of plane phase strains in rods and plates was performed in [5]. The solutions of the nonlinear axisymmetric problems given below were obtained using the kinematic shell model with independent rotations of transverse fibers [6].

Mechanical Equations of Axisymmetric Strain of a Shell. We consider a shell with axisymmetric base surface A. Let the material points of the deformable shell move relative to a cylindrical coordinate system (r, t_2, z) and let $\boldsymbol{i}_J$ be the orthonormal basis of this system. The local coordinate system t_J with the orthonormal basis $\boldsymbol{e}^0_J(t_1, t_2)$ is related to the base surface of the shell. The parameter $t_1 \in [0, 1]$ is reckoned along the meridian, $t_2 \in [0, 2\pi]$ along the parallel, and $t_3 \in [-1, +1]$ along the normal direction to the surface. Here and below, the upper-case Latin subscripts take the values 1, 2, and 3, the lower-case subscripts take the values 1 and 2, and a comma before a subscript denotes the partial derivative with respect to the corresponding coordinate.

In the undeformed state, the meridian of the base surface is specified by the parametric equations

$$r = la_2(t), \qquad z = la_3(t) \qquad \forall\, t \in [0, 1], \tag{1}$$

Institute of Computational Modeling, Siberian Division, Russian Academy of Sciences, Krasnoyarsk 660036; shkutin@icm.krasn.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 48, No. 2, pp. 163–171, March–April, 2007. Original article submitted February 20, 2006; revision submitted May 24, 2006.

where l is the meridional length and a_2 and a_3 are known functions of the parameter $t = t_1$. The orthonormal bases $\boldsymbol{i}_J$ and $\boldsymbol{e}_J^0$ can be related by the orthogonal transformation $\boldsymbol{e}_J^0 = \boldsymbol{i}_J \cdot O^0$ to the rotator tensor $O^0(t)$, whose components are defined in both bases by the orthogonal matrix

$$O_{JK}^0 = \begin{bmatrix} \cos\theta_0 & 0 & \sin\theta_0 \\ 0 & 1 & 0 \\ -\sin\theta_0 & 0 & \cos\theta_0 \end{bmatrix}, \tag{2}$$

where $\theta_0(t)$ is the angle of rotation of the basis $\boldsymbol{e}_J^0$ about the vector $\boldsymbol{e}_2^0 = \boldsymbol{i}_2$. By the definition, the relations $a_{2,1} = \cos\theta_0$ and $a_{3,1} = -\sin\theta_0$ are valid.

We study the axisymmetric strain of a dome for which its base surface remains axisymmetric and the equation of its meridian is written similarly to (1) in parametric form

$$r = ly_2(t), \qquad z = ly_3(t) \qquad \forall\, t \in [0,1], \tag{3}$$

where y_2 and y_3 are unknown functions of an arbitrary point t.

Using the orthogonal transformation $\boldsymbol{e}_J = \boldsymbol{i}_J \cdot O = \boldsymbol{e}_J^0 \cdot \overline{O^0} \cdot O$ with the rotator tensor $O(t)$, we introduce a local orthonormal basis $\boldsymbol{e}_J(t)$ which rotates during the deformation ($\overline{O^0}$ is the tensor conjugate with O^0). In the bases $\boldsymbol{i}_J$ and $\boldsymbol{e}_J$, the components of the rotator O are defined by a matrix of the form (2):

$$O_{JK} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}. \tag{4}$$

Here $\theta = \theta_0 + \vartheta$ is the angle of rotation of the basis $\boldsymbol{e}_J$ about the vector $\boldsymbol{i}_2$ and $\vartheta(t)$ is the rotation increment due to the strain. In the initial state, $\vartheta = 0$ and the basis $\boldsymbol{e}_J$ coincides with $\boldsymbol{e}_J^0$. The local rotation represented by the matrix (4) has only one degree of freedom — the angle of rotation θ or $\vartheta = \theta - \theta_0$.

To study the axisymmetric strains, we use the mechanical equations of the nonlinear shell model with independent fields of finite displacements and rotations [6, 7]. The starting system of equations comprises the kinematic relations

$$\begin{gathered} y_2' = (1+u_{11})\cos\theta + u_{13}\sin\theta, \qquad y_3' = -(1+u_{11})\sin\theta + u_{13}\cos\theta, \\ u_{22} = a_2^{-1}(y_2 - a_2), \qquad v_{11} = (\theta - \theta_0)', \qquad v_{22} = a_2^{-1}(\sin\theta - \sin\theta_0) \end{gathered} \tag{5}$$

and the dynamic (static) equations

$$\begin{gathered} (a_2T_1)' - T_{22} + a_2P_1 = 0, \qquad (a_2T_3)' + a_2P_3 = 0, \\ (a_2M_{11})' - M_{22}\cos\theta - a_2T_{13} + a_2Q_2 = 0, \\ T_{11} = T_1\cos\theta - T_3\sin\theta, \qquad T_{13} = T_1\sin\theta + T_3\cos\theta. \end{gathered} \tag{6}$$

Here $u_{iJ}(t)$, $v_{ii}(t)$, $T_{iJ}(t)$, and $M_{ii}(t)$ are the components of the metric and bending strains, forces, and moments, respectively, in the rotated basis and $T_1(t)$, $T_3(t)$, $P_1(t)$, $P_3(t)$, and $Q_2(t)$ are the components of the forces, external surface forces and moments in the cylindrical basis; the prime denotes differentiation with respect to t.

Micromechanical Constitutive Relations. The three-dimensional field of the axisymmetric strains of the shell is defined by the vectors $\boldsymbol{w}_1 = w_{11}\boldsymbol{e}_1 + w_{13}\boldsymbol{e}_3$ and $\boldsymbol{w}_2 = w_{22}\boldsymbol{e}_2$ and the stress field by the vectors $\boldsymbol{s}_1 = S_{11}\boldsymbol{e}_1 + S_{13}\boldsymbol{e}_3$ and $\boldsymbol{s}_2 = S_{22}\boldsymbol{e}_2$, where S_{11} and S_{22} are the tensile–compressive normal stresses and S_{13} is the transverse shear stress.

To establish the strain–stress relation in the direct phase transformation interval, we use the "uncoupled" [4] micromechanical constitutive relations

$$\begin{gathered} w_{11} = \varphi_{11} + \frac{S_{11} - \nu S_{22}}{E}, \quad w_{22} = \varphi_{22} + \frac{S_{22} - \nu S_{11}}{E}, \quad w_{13} = \varphi_{13} + \frac{S_{13}}{G}, \\ \frac{d\varphi_{11}}{dq} = \varkappa_0\varphi_{11} + \frac{2S_{11} - S_{22}}{3\sigma_0}, \quad \frac{d\varphi_{22}}{dq} = \varkappa_0\varphi_{22} + \frac{2S_{22} - S_{11}}{3\sigma_0}, \quad \frac{d\varphi_{13}}{dq} = \varkappa_0\varphi_{13} + \frac{S_{13}}{\sigma_0}, \\ q = \sin\Big(\frac{\pi}{2}\,\frac{T_+ - T}{T_+ - T_-}\Big), \qquad T_- \leqslant T \leqslant T_+, \qquad 0 \leqslant q \leqslant 1. \end{gathered} \tag{7}$$

Here w_{iJ} and φ_{iJ} are the total and phase strains, respectively, E and G are the tensile–compressive and shear elastic moduli, respectively, ν is Poisson's ratio, $\varkappa_0$ and σ_0 are experimental constants of the alloy in the direct-transformation interval, T_+ and T_- are the initial and final temperatures of the direct transformation, respectively, T is the current temperature, and q is the internal parameter of state treated as the volume fraction of the martensite phase. In the relations given above, the thermal strain of the alloy and the volume effect of the phase transformation are ignored; therefore, φ_{iJ} are the components of the phase-strain deviator. In addition, we assume that the phase transformation is a thermal process with a uniform temperature distribution over the volume of the specimen, which implies that the parameter q does not depend on the coordinates.

From (7) it follows that the phase strains are determined by differential (with respect to the parameter q) equations. The first terms on the right sides are responsible for the martensite crystal growth, and the second terms for the nucleation and orientation of the crystals in the direction of the acting stresses. The elastic moduli of the alloy appearing in (7) do not remain constant in the phase-transformation interval and vary from their austenite values to martensite values. Bearing in mind the meaning of the parameter q, we represent these moduli in the phase-transformation interval in the form of the Voigt-averaged relations

$$E = qE_- + (1-q)E_+, \qquad \nu = q\nu_- + (1-q)\nu_+, \qquad G = E/(2+2\nu),$$

where the subscripts minus and plus correspond to the martensite and austenite phases, respectively.

Assuming that the stresses depend on the parameter q more weakly than the phase strains, we obtain the approximate solution of the differential equations (7):

$$\varphi_{11} \simeq \eta\,\frac{2S_{11}-S_{22}}{3\sigma_0\varkappa_0}, \quad \varphi_{22} \simeq \eta\,\frac{2S_{22}-S_{11}}{3\sigma_0\varkappa_0}, \quad \varphi_{13} \simeq \eta\,\frac{S_{13}}{\sigma_0\varkappa_0}, \quad \eta(q) = \exp(\varkappa_0 q) - 1. \tag{8}$$

This solution satisfies the physical conditions: the phase strains vanish in the austenite (for $q = 0$) and reach maximum values in the martensite (for $q = 1$).

Substituting functions (8) into the first three equations of system (7), we obtain the approximate constitutive relations

$$\begin{gathered} E_0 w_{11} \simeq \eta_1 S_{11} - \eta_2 S_{22}, \quad E_0 w_{22} \simeq \eta_1 S_{22} - \eta_2 S_{11}, \quad E_0 w_{13} \simeq \eta_3 S_{13}, \\ \eta_1(q) \equiv \frac{E_0}{E} + \eta\,\frac{2E_0}{3\sigma_0\varkappa_0}, \quad \eta_2(q) \equiv \nu\,\frac{E_0}{E} + \eta\,\frac{E_0}{3\sigma_0\varkappa_0}, \quad \eta_3(q) \equiv \frac{E_0}{G} + \eta\,\frac{E_0}{\sigma_0\varkappa_0}. \end{gathered} \tag{9}$$

Here E_0 is a constant with the dimension of stress, which can conveniently be identified with one of the constants E_- or E_+.

Equations (9) describe the phase transformation as the thermoelastic strain with an implicit temperature dependence (through the parameter q).

In accordance with the shell model adopted, we have

$$w_{11} \simeq u_{11} + hl^{-1}t_3 v_{11}, \quad w_{22} \simeq u_{22} + hl^{-1}t_3 v_{22}, \quad w_{13} \simeq u_{13} \quad \forall\, t_3 \in [-1, +1]$$

($2h$ is the shell thickness). Substituting these values into (9) and integrating over the thickness, we obtain the resulting (moment) constitutive relations

$$\begin{gathered} C_0 u_{11} \simeq \eta_1 T_{11} - \eta_2 T_{22}, \quad C_0 u_{22} \simeq \eta_1 T_{22} - \eta_2 T_{11}, \quad C_0 u_{13} \simeq \eta_3 T_{13}, \\ l^{-1} H_0 v_{11} \simeq \eta_1 M_{11} - \eta_2 M_{22}, \qquad l^{-1} H_0 v_{22} \simeq \eta_1 M_{22} - \eta_2 M_{11}, \end{gathered} \tag{10}$$

where $C_0 = 2hE_0$ and $H_0 = 2h^3E_0/3$ are the stiffness parameters. To formulate the complete system of equations, it is convenient to write relations (10) as

$$\begin{gathered} u_{11} \simeq (1-\gamma^2)\eta_1 T_{11} C_0^{-1} - \gamma u_{22}, \qquad u_{13} \simeq \eta_3 T_{13} C_0^{-1}, \\ v_{11} \simeq (1-\gamma^2)\eta_1 M_{11} l H_0^{-1} - \gamma v_{22}, \\ T_{22} \simeq \gamma T_{11} + C_0 \eta_1^{-1} u_{11}, \quad M_{22} \simeq \gamma M_{11} + H_0 l^{-1} \eta_1^{-1} v_{11}, \quad \gamma = \eta_2 \eta_1^{-1}. \end{gathered} \tag{11}$$

Dimensionless Formulation of the Closed System of Equations. Equations (5), (6), and (11) constitute a closed system of ordinary nonlinear equations, whose solution depends on the parameter q. This system can be written in dimensionless form

$$y_0' = a_2^{-1}[(1-\gamma^2)\eta_1 y_1 - \gamma(\sin y_0 - \sin\theta_0)] + \theta_0',$$
$$y_1' = a_2^{-1}[\gamma y_1 + \eta_1^{-1}(\sin y_0 - \sin\theta_0)]\cos y_0 + \varepsilon^{-1} f_3 - a_2 q_2,$$
$$y_2' = a_2^{-1}\varepsilon\eta_3 f_3 \sin y_0 + (1+\varepsilon f_1)\cos y_0,$$
$$y_3' = a_2^{-1}\varepsilon\eta_3 f_3 \cos y_0 - (1+\varepsilon f_1)\sin y_0, \qquad (12)$$
$$y_4' = a_2^{-1}[\gamma f_2 + \varepsilon^{-1}\eta_1^{-1}(y_2 - a_2)] - a_2 p_1, \qquad y_5' = -a_2 p_3,$$
$$f_1 \equiv a_2^{-1}[(1-\gamma^2)\eta_1 f_2 - \varepsilon^{-1}\gamma(y_2 - a_2)],$$
$$f_2 \equiv y_4\cos y_0 - y_5 \sin y_0, \qquad f_3 \equiv y_4 \sin y_0 + y_5\cos y_0$$

for the unknown functions

$$y_0 = \theta, \quad y_1 = \frac{a_2 M_{11} l}{H_0}, \quad y_2 = \frac{r}{l}, \quad y_3 = \frac{z}{l}, \quad y_4 = \frac{a_2 T_1}{\varepsilon C_0}, \quad y_5 = \frac{a_2 T_3}{\varepsilon C_0}.$$

System (12) includes the parameters of state of the alloy η_1, η_2, η_3, and $\gamma = \eta_2/\eta_1$, the external loading parameters $p_1 = P_1 l/(\varepsilon C_0)$, $p_3 = P_3 l/(\varepsilon C_0)$, and $q_2 = Q_2 l^2/H_0$, and the geometrical shell parameter $\varepsilon^2 = h^2/(3l^2)$.

System (12) should be subject to boundary conditions formulated for particular problems. In the numerical solution of the system, its solutions are sought for discrete values of the parameter q in the interval $0 \leqslant q \leqslant 1$. The last equality in (7) relates the parameter q to the alloy temperature.

The solutions of the boundary-value problems given below were obtained for rods and plates made of NiTi (titanium nickelide) alloy with the following experimental parameter values of thermoelastic martensite transformation [3]: $T_- = 25°$C, $T_+ = 50°$C, $E_- = 28$ GPa, $E_+ = 84$ GPa, $E_0 = E_+$, $\sigma_0 = 0.049E_+$, $\varkappa_0 = 0.718$, $\nu_- = 0.48$, and $\nu_+ = 0.33$.

Buckling of a Plate under Uniform Pressure. We consider a simply supported circular plate loaded by a uniform normal pressure of intensity P in the austenite phase. The initial shape of the base surface (1) is specified by the parameters $\theta_0 = 0$, $z = 0$, and $r = lt$, where l is the radius of the supporting contour.

We study the axisymmetric strain of the plate (3) in the phase-transformation interval. In system (12), the surface-load components are specified by the functions $p_1 = p\sin y_0$, $p_3 = p\cos y_0$, and $q_2 = 0$, where $p = Pl/(\varepsilon C_0)$ is a numerical pressure parameter. On the supporting contour, the boundary conditions are given by

$$y_1(1) = 0, \qquad y_2(1) = 1, \qquad y_3(1) = 0. \qquad (13)$$

At the plate pole, the following boundary conditions formulated in terms of the primary unknown functions [7, 8] should be satisfied:

$$T_{13}(0) = 0, \qquad T_{11}(0) - T_{22}(0) = 0, \qquad M_{11}(0) - M_{22}(0) = 0, \qquad (14)$$

The numerical solution of the nonlinear boundary-value problem (12)–(14) obtained by the shooting method using the Mathcad software [7] is shown in Fig. 1 (w is the percent ratio of the maximum deflection to the radius). The points correspond to the evolution of the state parameter w in the phase-transformation interval for a fixed value of the pressure parameter and increasing value of q: in the austenite phase, the plate was loaded by a pressure $p = 0.01$ and then left to cool through the phase-transformation interval.

Figure 2 shows buckling modes (shapes of the meridian) of the plate for various values of the parameter q. Figure 3 shows the phase-strain evolution on the upper (free) surface of the plate for various values of the parameter q. One can see that the radial and circumferential strains reach maximum values at the pole and do not exceed 2%.

More detailed data on the phase-strain evolution of the plate are given in Table 1 for $p = 0.01$. The boundary values of the deflection w, rotation ϑ, strains $w_i = w_{ii}$ and $w_3 = w_{i3}$, internal force parameters $\tau_i = 100T_{ii}/(\varepsilon C_0)$, internal moments $\mu_i = 100M_{ii}l/H_0$, and stresses $s_i = 100S_{ii}/(\varepsilon E_0)$ and $s_3 = 100S_{i3}/(\varepsilon E_0)$ are given for some values of the parameter q. The tangential components of the strain and stress tensors reach maximum values at the plate pole, and transverse components (which are an order of magnitude smaller) reach maximum values on the supporting contour. One can see from Table 1 that with variation of the parameter q, the phase strains vary much faster than the stresses. This finding is consistent with the initial assumption of a weak dependence of the stress on the parameter q.

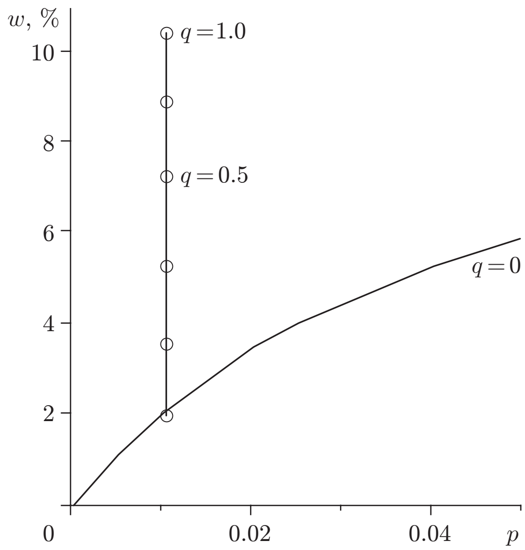

Fig. 1. Numerical solution of the deformation problem for a circular plate loaded by uniform pressure for $\varepsilon = 0.025$: the solid curve refers to the equilibrium states in the austenite phase with variation of the parameter p; the points show the evolution of the state parameter w for $p = 0.01$.

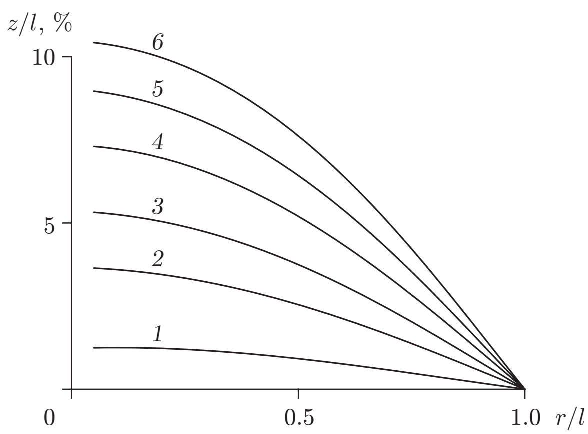

Fig. 2

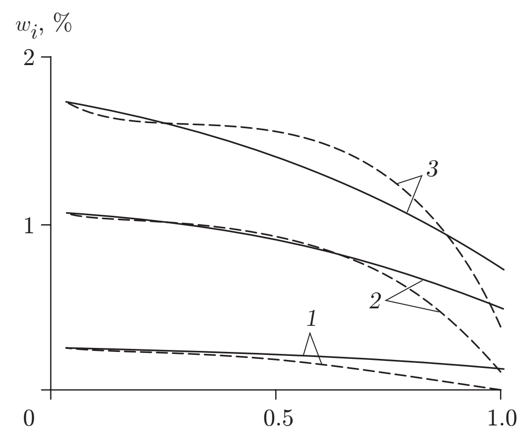

Fig. 3

Fig. 2. Buckling modes of the circular plate for $p = 0.01$ and $q = 0$ (1), 0.1 (2), 0.25 (3), 0.5 (4), 0.75 (5), and 1 (6).

Fig. 3. Evolution of the phase strains on the upper surface of the circular plate for $p = 0.01$ and $q = 0$ (1), 0.5 (2), and 1 (3): the solid curves refer to circumferential strains and the dashed curves refer to radial strains.

Buckling of a Spherical Dome under Uniform Pressure. The initial shape of the meridian of the dome (1) is defined by the parameters

$$\theta_0 = \alpha t, \qquad a_2 = \alpha^{-1} \sin\theta_0, \qquad a_3 = \alpha^{-1}(\cos\theta_0 - \cos\alpha),$$

where α is the slope of the meridian at the supporting point. The simply supported conditions of the dome are formulated by equalities of the form (13), where $y_2(1) = \alpha^{-1}\sin\alpha$. Conditions (14) remain unaltered. Figure 4 shows the numerical solution of this problem for $\varepsilon = 0.025$ (w is the maximum deflection normalized by the initial height of the dome).

Buckling modes of the dome for $p = 0.002$ and various values of the parameter q are shown in Fig. 5. The z coordinate is normalized by the height a and the r coordinate by the radius b of the supporting contour. Curve 1 ($q = 0.1$) is very close to the initial shape of the meridian and curve 4 ($q = 1$) refers to the everted state of the

TABLE 1

q	w, %	$\vartheta(1)$	$\tau_i(0)$	$\mu_i(0)$	$w_i(0)$, %	$w_3(1)$, %	$s_i(0)$	$s_3(1)$
0	2.03	0.0305	1.55	7.48	0.2428	0.0311	14.500	0.4677
0.10	3.61	0.0538	2.22	6.32	0.4633	0.0518	13.170	0.4144
0.25	5.28	0.0794	2.51	4.89	0.7197	0.0786	10.980	0.3543
0.50	7.28	0.1116	2.51	3.49	1.0667	0.1205	8.560	0.2924
0.75	8.91	0.1389	2.39	2.70	1.3890	0.1647	7.056	0.2529
1.00	10.40	0.1649	2.24	2.18	1.7313	0.2166	6.016	0.2237

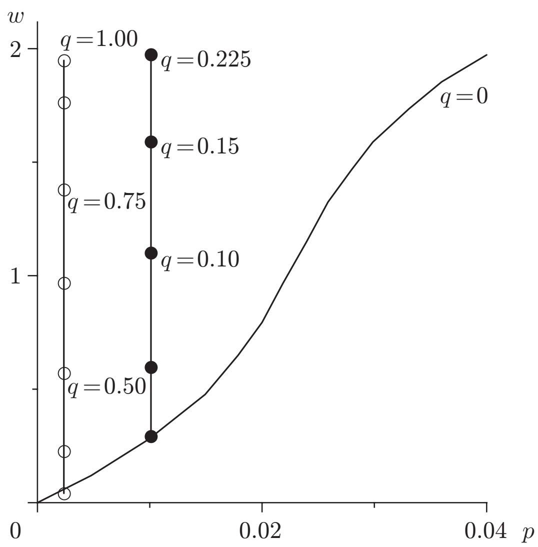

Fig. 4

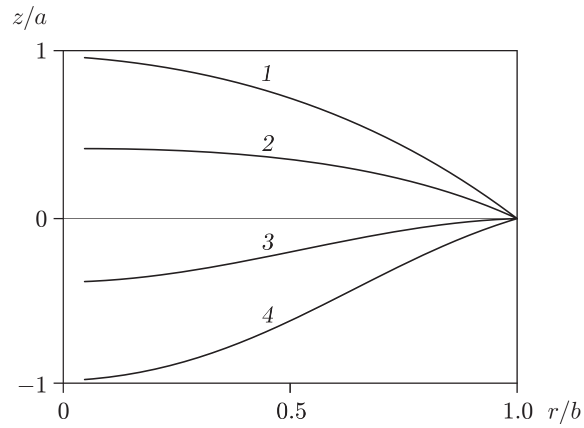

Fig. 5

Fig. 4. Numerical solution of the deformation problem for a spherical dome loaded by uniform pressure for $\varepsilon = 0.025$: the solid curve refers to equilibrium states in the austenite phase with variation of the parameter p; the points show the evolution of the state parameter w for a fixed value of the parameter p (the open points refer to $p = 0.002$ and the filled points refer to $p = 0.01$).

Fig. 5. Buckling modes of a spherical dome for $p = 0.002$ and $q = 0.1$ (1), 0.5 (2), 0.75 (3), and 1 (4).

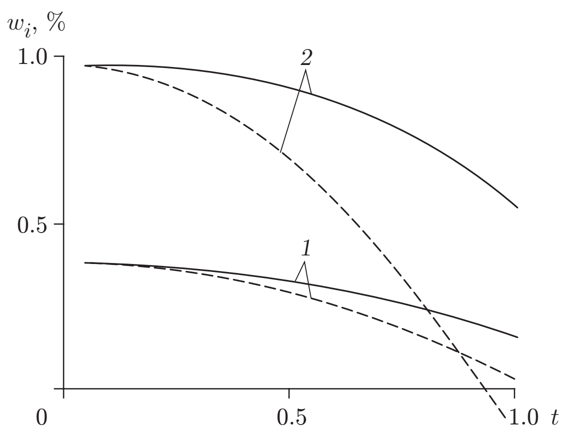

Fig. 6. Phase-strain evolution for the upper surface of the spherical dome for $p = 0.002$ and $q = 0.5$ (1) and 1 (2): the solid curves refer to circumferential strains and the dashed curves to meridional strains.

TABLE 2

q	w	$\vartheta(1)$	$\tau_i(0)$	$\mu_i(0)$	$w_i(0)$, %	$w_3(1)$, %	$s_i(0)$	$s_3(1)$
0	0.0514	0.0034	0.7314	0.8090	0.0357	0.0037	2.133	0.0557
0.10	0.1156	0.0075	0.7227	0.8832	0.0792	0.0071	2.252	0.0565
0.25	0.2374	0.0152	0.7333	0.9860	0.1600	0.0132	2.441	0.0595
0.50	0.5814	0.0365	0.7570	1.2976	0.3744	0.0288	3.005	0.0700
0.75	1.3976	0.0869	0.5137	2.0046	0.7846	0.0649	3.986	0.0997
1.00	1.9554	0.1233	0.0781	1.8824	0.9609	0.1006	3.338	0.1039

dome. The dome is everted without snap-through buckling since $w(q)$ is a monotonically increasing curve [like the curve of $w(p)$].

Figure 6 shows the phase-strain evolution on the upper (stress-free) surface of the dome for various values of the parameter q. The meridional and circumferential strains as functions of the coordinate t are shown by dashed and solid curves, respectively. One can see that the maximum values of the meridional and circumferential strains occur at the pole and do not exceed 1%.

More detailed data on the phase-strain evolution of the dome are listed in Table 2 for $p = 0.002$. The absolute boundary values of the deflection w, rotation ϑ, strains w_i, internal-force parameters τ_i, internal moments μ_i, and stresses s_i are given for some values of q. As in the plate problem, the tangential components of the strain and stress tensors reach maximum values at the pole and the transverse components (which are an order of magnitude smaller) reach maximum values on the supporting contour; in this case, the phase strains are more sensitive to the parameter q than the stresses.

Conclusions. The solutions given above should be regarded as approximate solutions in which interphase stress rates are neglected compared to the phase-strain rates in the phase-transformation temperature interval. This solution is supported by the presence of "yield plateaus" in the experimental curves of stresses versus phase strains.

The formulation of equations proposed here allows one to formulate and approximately solve strongly nonlinear axisymmetric problems of phase strains in thin shells and plates under other loading and boundary conditions.

This work was supported by the Russian Foundation for Basic Research (Grant No. 04-01-00267).

REFERENCES

1. G. V. Kurdyumov and L. G. Khandros, "On the 'thermoelastic' equilibrium and martensite transformations," *Dokl. Akad. Nauk SSSR*, **66**, No. 2, 211–214 (1949).
2. A. A. Movchan, "Micromechanical constitutive equations for shape-memory alloys," *Probl. Mashinost. Nadezh. Mashin*, No. 6, 47–53 (1994).
3. A. A. Movchan, "The effect of variable elastic moduli and stresses on the phase content of shape-memory alloys," *Izv. Ross. Akad. Nauk. Mekh. Tverd. Tela*, No. 1, 79–90 (1998).
4. A. A. Movchan and L. G. Sil'chenko, "Stability of a plate from a shape-memory alloy in direct thermoelastic phase transformation," *Prikl. Mat. Mekh.*, **68**, No. 1, 83–90 (2004).
5. L. I. Shkutin, Analysis of plane phase strains of rods and plates," *J. Appl. Mech. Tech. Phys.*, **47**, No. 2, 282–288 (2006).
6. L. I. Shkutin, "Incremental deformation model for a rod," *J. Appl. Mech. Tech. Phys.*, **40**, No. 5, 7557–762 (1999).
7. L. I. Shkutin, "Numerical analysis of axisymmetric buckling of conical shells," *J. Appl. Mech. Tech. Phys.*, **42**, No. 6, 1057–1063 (2001).
8. L. I. Shkutin, "Numerical analysis of axisymmetric buckling of plates under radial compression," *J. Appl. Mech. Tech. Phys.*, **45**, No. 1, 89–95 (2004).